The amazing powers of Generalized Moving Least-Squares

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Generalized Moving Least-Squares (GMLS) is a powerful non-parametric regression technique that allows you to approximate any bounded linear functional from scattered samples of its argument. While GMLS has its origins in classic scattered data approximation theory in this talk I will show that it can be a surprisingly effective and versatile tool for the numerical solution of PDEs. To that end I will examine three different schemes developed by our team at Sandia, that take advantage of the amazing properties of GMLS:

- A staggered meshfree scheme that mimics locally properties of classical schemes such as MAC and Finite Volumes in a meshfree setting;
- A Meshfree Mimetic Divergence operator that satisfies a discrete divergence theorem and can be used to construct a conservative meshfree (virtual) finite volume scheme;
- A hybrid Discontinuous Galerkin method that uses a point cloud to define robust "shape" functions whose quality does not depend on the quailty of the underlying mesh, which is used only to integrate the weak forms.

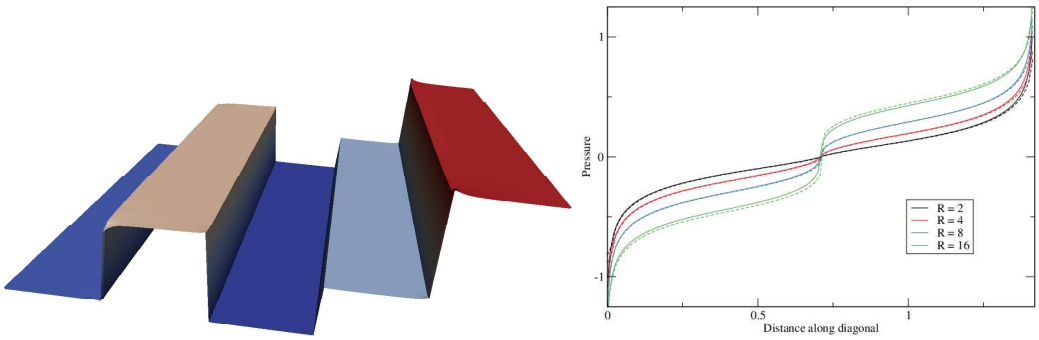

Figure 1: *Left:* Approximation of the horizontal flux component in the five strip benchmark by the virtual finite volume scheme is undistinguishable from that by a mixed method using RT0-P0 elements. *Right:* Comparison of the pressure profiles of the five spot benchmark along $y = x$ by the virtual finite volume scheme (solid lines) and a mixed method using RT0-P0 elements (dashed lines) for increasing diffusivity ratios $R = \varepsilon_1/\varepsilon_2$.

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